Super Conductive Levitation Based on the Magnetic Potential Well Phenomenon

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ABSTRACT: The Magnetic Potential Well (MPW) as the ability of two-magnet force to change attraction into repulsion when only spacing between two removed magnets decreases is re-declared. It is first proved on a simple one-dimensional model. After that, using the MPW-condition and parameter restrictions resulting in the potential energy minimum, a dynamic system with six degrees of freedom is considered. This approach and computer algebra software Maple is used to substantiate the stability of a free vehicle magnetically suspended by two-wire dc line and to analyze the vehicle's dynamics. Some tested results are represented.

1 INTRODUCTION

1.1 Known static levitation effects

There are several physical mechanisms that can cause a body to float freely [1]. As regard passive magnetic levitation phenomena, Gilbert [5] examined this problem as early as 1600. In 1842 Earnshaw [4] showed that free particles governed by inverse square law forces cannot be in the stable equilibrium. This result is usually considered as a fundamental instability principle for equilibrated magnetic systems. After Earnshaw nothing remains but to search exceptions to this principle. The diamagnetism of a substance was the first exception to be found by the German physicist Braunbeck [2, 3]. He corroborated this conclusion by testing and anticipated modern Meissner repulsive super conductive levitation. In 1976 the MPW phenomenon was found [11]. Additionally, MPW-suspension theory was developed and corroborating tests, particularly suspension of two free niobium rings below a rear-earth permanent magnet were carried out. Later the suspension levitation was re-derived using high-temperature superconductors (see e.g. [8]).

1.2 Dynamic levitation effects

Free particles interacting by gravity, electrical or magnetic forces are similar with respect to their equilibrium stability: on the base of Earnshaw's principle they are, as a rule, unstable. Dynamic planetary motions in gravity and electrostatic fields (without energy dissipation) can be stable. Magnetic planetary systems essentially vary from gravitational and electric ones by potential energy law and stability. If potential energy of two point masses or electrical charges is proportional to $1/R$ (R = distance), for two magnetic dipoles it is proportional to $1/R^3$. As showed by Tamm [15], motions in this case correspond to falling on an attractive center. Later Ginzburg [6] tried to derive solution of the "$1/R^3$ problem" taking into account a particle's space extent by introducing the form-factor. Another approach was proposed in [9, 10] where stable magnetic planetary systems were first revealed.

Other magnetic dynamic systems e.g. based on sign-variable force effect [7], active feedback control, or electrodynamic repulsive effect [14] can be stable and find themselves in wide applications.

2 SMALL LOOP AND TWO-WIRE DC LINE

2.1 Magnetic potential energy

The MPW-theory was first studied in [11]. A magnet in [11] is modeled by a closed loop of zero electrical resistance whose longitudinal size is much greater than the loop thickness. The theory takes into account variability of electrical currents caused by frozenness of the full magnetic fluxes (magnetic linkages) in the loops. The individual inductance and the mutual inductance (a function of spacing between the loops) are expressed as double linear integrals in terms of inverse distances. To determine inductances in our case (one loop is infinitesimal as compared with two-wire line) it is more convenient to use the well-known magnetic induction formula for the rectilinear electric current magnetic field. If a line plane is horizontal and Oz is vertical axis of its symmetry where a small plane closed super conductive loop is horizontally placed and z–removed from the line, at points of the loop placing modulus of the vertical component of magnetic induction can be written as

$$B_z = \frac{\mu_0 I}{\pi} \cdot \frac{a}{a^2 + z^2} \tag{1}$$

where I = line current, $2a$ = line width, and supposition about homogeneity of the line magnetic field at points of the loop disposition is assumed. Then the line-loop mutual inductance L_{12} is

$$L_{12} = \frac{\mu_0 S}{a\pi} \cdot \frac{1}{1 + x^2} \tag{2}$$

where S = loop area and non-dimensional distance $x = z/a$ is introduced.

Let us use the magnetic force formula (see [11], formula (4.8))

$$P = \frac{\Psi_1 L_{22} - \Psi_2 L_{12}}{\left(L_{11} L_{22} - l_{12}^2\right)^2} (\Psi_2 L_{11} - \Psi_1 L_{12}) \frac{\partial L_{12}}{\partial z} \tag{3}$$

where L_{11} and L_{22} are individual inductances of magnetically interacting loops, $L_{12}(z)$ is their mutual inductance, and Ψ_1, Ψ_2 are magnetic linkages of the loops. If index 2 is applied to a two-wire line of infinite length, parameters Ψ_2 and L_{22} tend to infinity. In this case, formula (3) can be transformed to the form

$$P \approx \frac{\Psi_2^2}{L_{22}^2 L_{11}} [L_{12}(0) - L_{12}(z)] \frac{\partial L_{12}(z)}{\partial z} \tag{4}$$

where $L_{12}(0) = L_{22}\Psi_1 / \Psi_2$ is the line-loop mutual inductance at $z = z_0$ (z_0 is the MPW-position at which magnetic force between line and loop is zero).

The potential energy of magnetic interaction corresponding to the force-law (4) is

$$U = \frac{\Psi_2^2}{L_{22}^2 L_{11}} [L_{12}(0) - L_{12}(z)]^2 \cdot \tag{5}$$

This expression can be applied to any pair of magnetically interacting loops with a great difference in their longitudinal sizes. It holds true in the case when mutual inductance depends on more than one coordinate z. Then instead of the simplest case $L_{12}(z)$, the function $L_{12}(q_i)$, $i = 1, 2, \ldots, n$ must be considered. It should be noted that we knowingly do not replace the ratio Ψ_2^2 / L_{22}^2 by the line current squared I^2 because magnetic energy can be considered as the potential one only when it is represented in terms of linkages and mechanical coordinates (see [16], chapter 1).

Expressions (2) and (5) after simple transformations and omitting multipliers unessential in our problem give non-dimensional magnetic potential energy

$$u = \left[1/(1 + x_0^2) - 1/(1 + x^2)\right]^2 \tag{6}$$

where $-\infty < x, x_0 < +\infty$ are non-dimensional distances between loop and line. One of them (x) is an actual spacing at which force characteristics are subject of interest. Another distance (x_0) is a constant parameter, a subject of our discretion.

2.2 MPW- conditions and vertical stability

Maple-operations [12] with (6) allow improved force parameter calculations. Below we show some of them in order to derive the spacing range inside of which the magnetic attractive force increases together with increasing in spacing. This condition corresponds with positive rigidity c i.e. vertical stability of the MPW-suspension. Spacing of the MPW- levitation must be in the range between x_0 and x, determined by the curve in Figure 1. Explicit forms for the non-dimensional force p and rigidity c can be derived if colons below are replaced by semicolons. In a similar way plots of p and c can be obtained.

```
> restart;
> u:=(1/(1+x0^2)-1/(1+x^2))^2:
> p:=-diff(u,x):
> c:=diff(p,x):
> simplify(c):
>x0:=x*(3*(x^2-1)/(5*x^2-1))^(1/2):
> plot(x0,x=1..2);
```

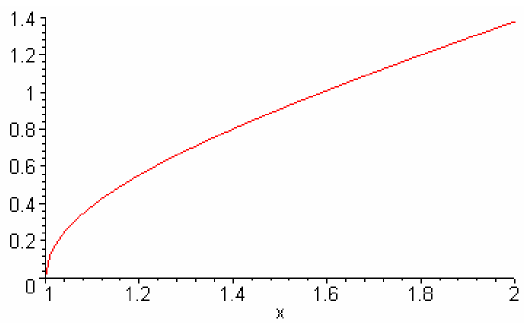

Figure 1: MPW-position (x_0) versus maximal spacing (x) allowing vertical stability of the MPW-suspension

In accordance with the MPW-theory [11], at MPW-position the mutual inductance equals the product of the linkages ratio less than unity and the self-inductance of a loop with higher linkage. There are some practicable ways to realize MPW-position at some spacing x_0. One of them consists in the following: zero resistance of a part of the small loop placed horizontally at spacing x_0 must be destroyed for certain time and then restored in the presence of the line current I.

3 STABILITY WITH MPW- SUSPENSION

3.1 Potential energy

In Figure 2 the immobile coordinate system *Oxyz* with unit vectors $\vec{i}_1, \vec{i}_2, \vec{i}_3$ and rigidly connected with a free vehicle system $O_1 x'y'z'$ with unit vectors $\vec{i}_1', \vec{i}_2', \vec{i}_3'$ are shown. Opposite directed currents I of a dc two-wire line are parallel to axis *Ox*. On the vehicle top in parallel to the plane $O_1 x'y'$ two small super conductive closed loops are symmetrically placed. The vehicle mass center O_1 shifted with pendulum factor L in the $O_1 z'$ direction is described by three Cartesian coordinates x, y, z (dimensional) or $x_1 = x / A_1$, $x_2 = y / A_1$, $x_3 = z / A_1$ (non-dimensional) where A_1 is the half distance between loops. Space orientation of the vehicle is described by three angles x_4, x_5, x_6 determining roll, pitch, and yaw respectively. The vehicle weight G is directed downward opposite to axis *Oz*.

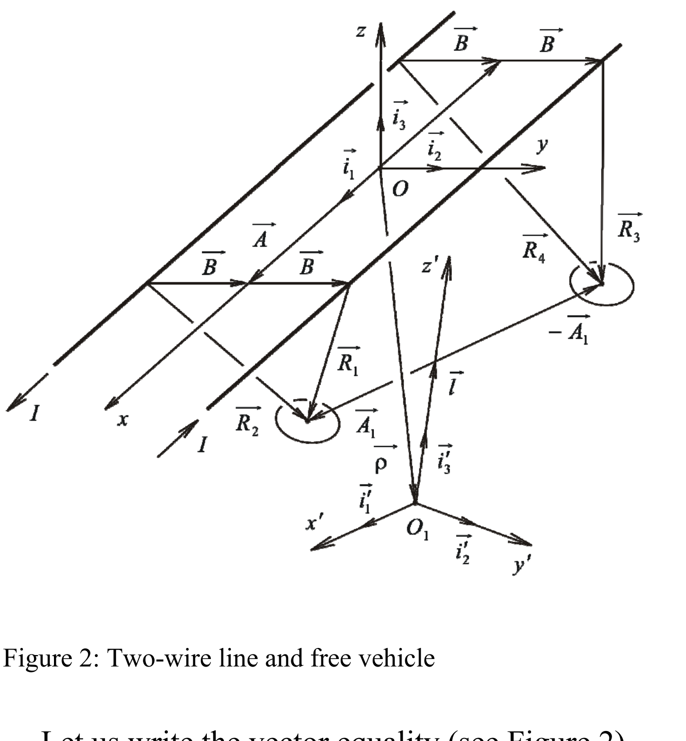

Figure 2: Two-wire line and free vehicle

Let us write the vector equality (see Figure 2)

$$\vec{R}_1 = \vec{\rho} + \vec{l} + \vec{A}_1 - \vec{B} - \vec{A} \tag{7}$$

where

$$\vec{\rho} = x\vec{i}_1 + y\vec{i}_2 + z\vec{i}_3, \quad \vec{l} = L\vec{i}_3', \quad \vec{A}_1 = A_1\vec{i}_1',$$
$$\vec{R}_1 = R_{12}\vec{i}_2 + R_{13}\vec{i}_3, \quad \vec{B} = B\vec{i}_2, \quad \vec{A} = A\vec{i}_1. \tag{8}$$

Similar expressions for other vectors of distances $\vec{R}_2, \vec{R}_3, \vec{R}_4$ between loops and currents can be written.

Selecting A to guarantee zero Ox–component for the vector $\vec{R}_1$, using matrix of direction cosines between unit vectors introduced above

$$\begin{array}{cccc} & \vec{i}_1 & \vec{i}_2 & \vec{i}_3 \\ \vec{i}_1' & a_{11} & a_{12} & a_{13} \\ \vec{i}_2' & a_{21} & a_{22} & a_{23} \\ \vec{i}_3' & a_{31} & a_{32} & a_{33}, \end{array} \tag{9}$$

on the base of (5) it is possible to derive the potential energy

$$U = K\left[M - \left(\frac{r_{13}}{r_1} - \frac{r_{23}}{r_2}\right)a_{32} - \left(-\frac{r_{12}}{r_1} + \frac{r_{22}}{r_2}\right)a_{33}\right]^2 + \tag{10}$$
$$+ K\left[M - \left(\frac{r_{33}}{r_3} - \frac{r_{43}}{r_4}\right)a_{32} - \left(-\frac{r_{32}}{r_3} + \frac{r_{42}}{r_4}\right)a_{33}\right]^2 + GA_1x_3$$

where

$$K = \frac{\Psi_2^2}{2L_{11}L_{22}^2}\left(\frac{\mu_0 S}{4\pi A_1}\right)^2, \; r_i = r_{i2}^2 + r_{i3}^2, \; i = 1, \ldots, 4$$

$$r_{12} = x_2 + la_{32} + a_{12} - b, \quad r_{22} = x_2 + la_{32} + a_{12} + b,$$
$$r_{13} = x_3 + la_{33} + a_{13} = r_{23},$$
$$r_{32} = x_2 + la_{32} - a_{12} - b, \quad r_{42} = x_2 + la_{32} - a_{12} + b, \tag{11}$$
$$r_{33} = x_3 + la_{33} - a_{13} = r_{43},$$
$$M = 2b/(b^2 + (x_3 + b)^2), \quad l = L/A_1, \quad b = B/A_1.$$

3.2 Stability conditions

A Maple-program to derive the Taylor series for the potential energy (10) using presentations of direction cosines [13] can be

```
> restart;
> a11:=cos(x5)*cos(x6):
> a12:=-cos(x5)*sin(x6): a13:=sin(x5):
>a21:=sin(x4)*sin(x5)*cos(x6)+cos(x4)*si
n(x6):
>a22:=-
sin(x4)*sin(x5)*sin(x6)+cos(x4)*cos(x6):
a23:=-sin(x4)*cos(x5):
>a31:=-
cos(x4)*sin(x5)*cos(x6)+sin(x4)*sin(x6):
>a32:=cos(x4)*sin(x5)*sin(x6)+sin(x4)*co
s(x6): a33:=cos(x4)*cos(x5):
> r12:=x2+l*a32+a12-b:
> r22:=x2+l*a32+a12+b:
> r13:=x3+l*a33+a13:
> r32:=x2+l*a32-a12-b:
```

```
> r42:=x2+l*a32-a12+b:
> r33:=x3+l*a33-a13:
> r1:=r12^2+r13^2: r2:=r22^2+r13^2:
> r3:=r32^2+r33^2: r4:=r42^2+r33^2:
>u:=(M-(1/r1-1/r2)*r13*a32+(r12/r1-
r22/r2)*a33)^2+(M-(1/r3-
1/r4)*r33*a32+(r32/r3-
r42/r4)*a33)^2+k1*x3:
> U:=mtaylor(f,[x2,x4,x5,x6],3):
```

The last Maple-expression is the desired series. It is very cumbersome to be recorded here in the explicit form. Simplifying results in

$$U = U_0 + U_1 x_3 + u_{22} x_2^2 + u_{24} x_2 x_4 + u_{44} x_4^2 + \\ + u_{55} x_5^2 + u_{66} x_6^2 \qquad (12)$$

where half U_0 is the magnetic potential energy of the small loop and two-wire line at coaxial positions (unperturbed motion when perturbations x_2, x_4, x_5, x_6 are zero). Two first terms from the right of (12) are responsible for the vertical dynamics (i.e. the MPW-manifestation direction considered above). The rest is a quadratic form responsible for the stability of a free MPW-levitated vehicle (equilibrated or moved along axis Ox) relative to x_2, x_4, x_5, and x_6.

The stability problem was Maple-investigated in approximation $|X| = |x_3 + l| << 1$ (small levitation gaps relative to distance between loops).

Maple–investigation of when the above quadratic form is positive that is equivalent to the stability sufficient conditions for the unperturbed motion results in:

1. Guaranteeing the lateral stability and stability with respect to yaw-turning requires that the non-dimensional gap must be more than 0.408 of the non-dimensional half line's width

$$|X| > b\sqrt{1/6} \approx 0.408 \cdot b \,. \qquad (13)$$

2. The roll-stability requires satisfying the inequality

$$|X| > \frac{2l^2 b^2}{1+6b^2} \approx 2l^2 b^2 . \qquad (14)$$

At small b and l this inequality is less strong than (13).

3. The orientation-lateral stability requires the inequality $5X^2 + 2X \cdot l - b^2 > 0$ must be satisfied. At small b and l this condition amounts to

$$X < -2l/5 \,. \qquad (15)$$

4. The pitch-turning stability depends on pendulum factor that assists stability and other geometrical parameters. Excluding pendulum factor influence, we derive the condition

$$|X| > \sqrt{b/6} \approx 0.408\sqrt{b} \,. \qquad (16)$$

Thus, if the levitation gap, pendulum factor, and line's width are small relative to the distance between vehicle's loops, the orientation-lateral part of the levitation stability is determined by restrictions on the levitation gap that must be complied with the derived inequalities. Additionally, it is necessary to satisfy stability conditions in the vertical direction (MPW-suspension direction): the levitation gap $|X|$ must be inside the range $x_0 < |X| < x$ where x is determined by the curve in Figure 1 (at chosen x_0). And finally, it is necessary to suffice for the magnetic attractive force as equilibrant of the vehicle weight at the unperturbed motion.

4 THE FREE VEHICLE DYNAMICS

4.1 Equations

Formula (12) shows that on assumption of small perturbations, potential energy depends on four degrees of freedom of the free vehicle: lateral displacement x_2, angles of roll x_4, pith x_5, and yaw x_6. Parameter x_3 in the Taylor series (12) must be considered as a constant satisfying equilibration condition. Thus, the free vehicle motion decomposes so that movements in the vertical direction (oscillations) and along two-wire line (inertial motion) do not influence on other coordinates dynamic behavior. Therefore, it is pertinent to study interrelated lateral and orientation motions of the free vehicle.

After above remarks the non-dimensional form of the orientation-lateral dynamic subsystem can be as

$$\frac{d^2 x_2}{dt^2} = -b_2 x_2 - b_{24} x_4 , \quad \frac{dn_1}{dt} = -b_{42} x_2 - b_4 x_4 ,$$

$$\frac{dn_2}{dt} = \frac{C_2 - C_1}{C_1} n_3 n_1 - b_5 x_5 , \quad \frac{dn_3}{dt} = \frac{C_1 - C_2}{C_1} n_1 n_2 - b_6 x_6 ,$$

$$n_1 = \frac{dx_4}{dt} + x_6 \cdot \frac{dx_5}{dt} , \quad n_2 = -x_6 \cdot \frac{dx_4}{dt} + \frac{dx_5}{dt} , \qquad (17)$$

$$n_3 = x_5 \cdot \frac{dx_4}{dt} + \frac{dx_6}{dt} .$$

Here C_1 and C_2 are the vehicle's central inertia moments with respect to long and transverse axes respectively; b_2, b_4, b_{24}, b_{42}, b_5, and b_6 are constant non-dimensional rigidities coefficients; n_1, n_2, and n_3 are non-dimensional vehicle's angular velocity components on axes $O_1 x'$, $O_1 y'$, $O_1 z'$ respectively, and t is non-dimensional time.

4.2 Dynamics analysis

Maple allows us to derive solutions for the system (17) and its phase portraits. The numerical values of parameters and the initial conditions are clear from Maple-commands below

```
>restart;with(plots):e1:=diff(x2(t),t)=y2(t);e2:=diff(y2(t),t)=-b2*x2(t)+b24*x4(t);e3:=diff(n1(t),t)=-b4*x4(t)+b42*x2(t);e4:=diff(n2(t),t)=n3(t)*n1(t)-b5*x5(t);e5:=diff(n3(t),t)=-n1(t)*n2(t)-b6*x6(t);e6:=diff(x4(t),t)+x6(t)*diff(x5(t),t)=n1(t);e7:=-x6(t)*diff(x4(t),t)+diff(x5(t),t)=n2(t);e8:=x5(t)*diff(x4(t),t)+diff(x6(t),t)=n3(t);
> b2:=10:b24:=0.5:b4:=2:b42:=5:b5:=40:b6:=1:
>s:=dsolve({e1,e2,e3,e4,e5,e6,e7,e8,x2(0)=0.1,y2(0)=0,x4(0)=0.2,x5(0)=0.05,x6(0)=0.1,n1(0)=0,n2(0)=0,n3(0)=0},{x2(t),y2(t),x4(t),x5(t),x6(t),n1(t),n2(t),n3(t)},type=numeric):
>odeplot(s,[[t,x4(t)],[t,x6(t)]],0..30,numpoints=500,color=black);
```

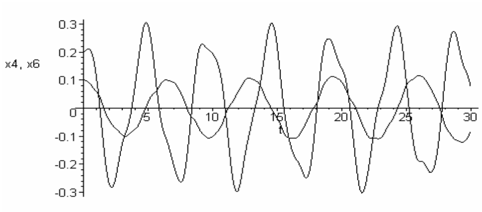

Figure 3: Solutions for roll ($x4$) and yaw ($x6$) angles

```
>odeplot(s,[y2(t),x4(t)],0..25,numpoints=500,color=black);
```

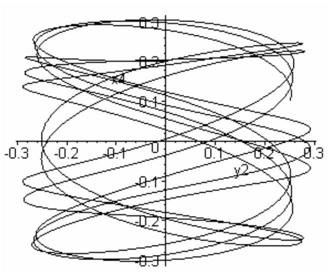

Figure 4: Lateral speed ($y2$)-roll angle ($x4$) phase portrait

```
>odeplot(s,[x2(t),y2(t),x4(t)],0..30,numpoints=500,axes=BOX,color=black);
```

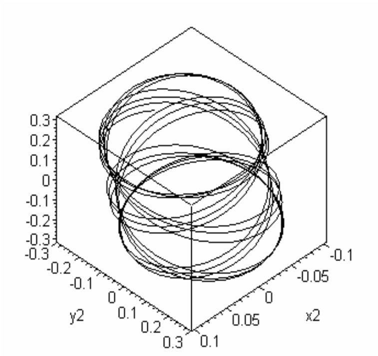

Figure 5: Lateral displacement ($x2$)-lateral speed ($y2$)-roll angle ($x4$) phase portrait

```
>odeplot(s,[t,x2(t),x4(t)],0..30,numpoints=500,axes=BOX,color=black);
```

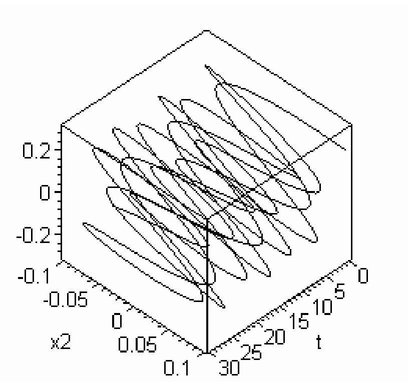

Figure 6: Lateral–roll displacements development in time

Thus, the developed free vehicle dynamic model allows deriving solutions for a complicated system of non-linear differential equations. It also displays the influence of geometrical and magnetic parameters on the dynamics of a free vehicle. Moreover, this approach shows the power of Maple to study different current carrying loop configurations and parameter values that could be used to determine distinguished features of levitated dynamic systems based on the MPW-phenomenon.

5 SOME MPW-TESTS

5.1 MPW nearby the critical current mode

First tests reduced to attesting the MPW-existence and MPW-suspension of two free niobium rings in the field of a rare-earth permanent magnet [11].

In the light of practical applications it is important to estimate the MPW-force maximum. This problem was dealt with by measuring magnetic forces between three coaxial coils when two constraints were satisfied: one, existence of the MPW; and two, existence of current conditions approaching the critical mode of one of the coils. Three niobium-titanium coils were wound. Two smaller coils (30 mm in outside diameter and 3 × 3 mm^2 in cross-section) were movable and a third larger one (60 mm in outside diameter and 5 × 5

mm^2 in cross-section) was immobile. Before force measurements, the two smaller coils were kept close against one another and were placed symmetrically inside the larger coil and transferred into resistive state by thermal keys. Under these conditions the critical current I_c of the larger coil was determined. Subsequently, the larger coil was fed by electrical current a little less than I_c and first the larger coil and then two smaller ones were transferred into the persistent current mode. Thereupon, two smaller coils were moved apart symmetrically and owing to this they formed a gap h. Subsequently the gap $h = 4$ mm was held fixed. Finally, the pair of smaller coils was x-displaced and magnetic force P was measured as a function of the displacement x. Measurements demonstrated the increase in the magnetic attractive force P from zero at $x = 2$ mm to the maximum of 900 N at $x = 5$ mm and the destruction of superconductivity after attaining the magnetic attractive force maximum.

5.2 Side-wall magnet configuration

MPW-tests were also conducted with a side wall levitation configuration consisted of three identical super conductive coils (of 600 mm in inside diameter, 100 mm in height) with a small ratio between thickness and diameter.

The individual metal cryostat was fabricated for each coil. They were of a cylindrical form with the horizontal-working axis. The coil was located at the bottom of its cryostat while the helium and nitrogen volumes were at the top. Detachable cooled input terminals, the maximum effective current was 300 A, were incorporated in the design together with a unit to transfer horizontal and vertical forces up to 10,000 N and 20,000 N respectively.

A distance separating two outer super conductive magnets could be varied so axial warm spacing between the movable magnet and fixed ones could be also varied over a range of 0 mm to 200 mm.

Two outer super conductive magnets were mounted to a frame by screws. The frame, bodies and main details of all super conductive magnets were manufactured from stainless steel.

During testing the movable magnet was held, lifted, and lowered into the space between outer magnets by an elevating crane. A platform with lead ingots of known masses measuring vertical magnetic forces between movable and outer magnets at different vertical and fixed horizontal warm spacing was attached to the movable magnet bottom.

Upon cooling, all super conductive magnets were fed by individual low voltage sources. Foam plastic and wood spacers were used to prevent possible bumps of the movable magnet against outer ones. Spacers were removed before magnetic force measurements.

The major result of these experiments was the vertically stable MPW-suspension of 750 kg mass at warm vertical and lateral spacing of 300 mm and 120 mm respectively. Super conductive magnets operated in the persistent current mode.

6 REFERENCES

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